

## Quantitative Internship Candidate Test

31st March 2017

### Problem 1

- a) If  $X$  and  $Y$  are two independent uniform random numbers on the interval  $[0, 1]$ , what is the probability density of  $X + Y$ ?
- b) Suppose we know that  $Z$  is distributed according to the distribution whose probability density is

$$p(x) = \frac{1}{\pi(1+x^2)}.$$

What is the mean and the variance of the distribution? What would be the undiscounted price of a vanilla payoff  $(x - k)^+$  with this distribution? Explain your findings.

### Problem 2

You are playing a game where you can throw a die up to three times. You will earn the face value of the *last* throw, i.e. 1 to 6 euros. You have the option to stop after each throw and walk away with the money earned. What is the expected payoff of this game and what is the optimal strategy to follow?

### Problem 3

Write an implementation in your favourite programming language for the function that evaluates  $|\sin(x)/x|$  in the  $x \in [-3, 3]$ . Pay attention to the region close to zero!

### Problem 4

Write a small root-finder program in your favourite language that finds the root of  $f(x) = x^3 + 2x^2 - 3x - 2$  in the  $x \in [0, 2]$  interval.

### Problem 5

- a) What are Cash-or-Nothing digital options and how can they be replicated with vanilla options? Please generate a plot of the delta and vega profile of the digital at inception and at expiry?
- b) What is the difference between lending with or without collateral? In which case will the interest rate charged be higher and why?
- c) Assume company A has taken a loan from company B where the quarterly interest is EURIBOR 3M+1%. Which financial instrument could be used to get something that would guarantee fixed payments (and hence company A would not depend on changes in the EURIBOR rate any more)? Please explain all cashflows for party A and party B.